

FIGURE 8.40

Cascade realization of IIR filter in Example 8.23 in direct-form II.

The difference equations for the implementation of the first section are

$$w_1(n) = x(n) - 0.5654w_1(n-1) - 0.4776w_1(n-2)$$

$$y_1(n) = 0.5108w_1(n) + 1.0215w_1(n-1) + 0.5108w_1(n-2)$$

The difference equations for the implementation of the second section are

$$w_2(n) = y_1(n) - 0.4129w_2(n-1) - 0.0790w_2(n-2)$$

$$y(n) = 0.3730w_2(n) + 0.7460w_2(n-1) + 0.3730w_2(n-2)$$

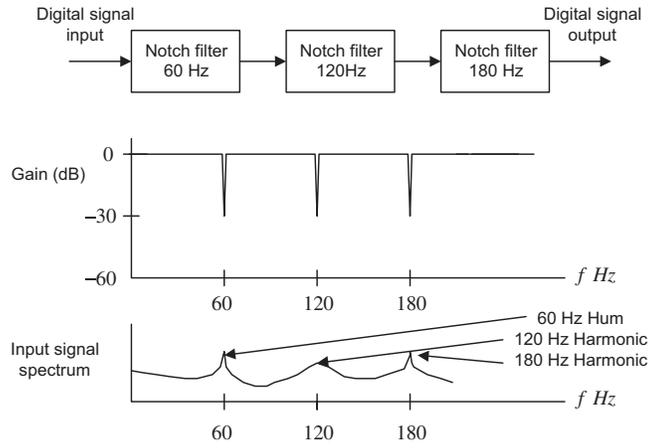
Note that for both direct-form I and direct-form II, the output from the first filter section becomes the input for the second filter section.

## 8.9 APPLICATION: 60-HZ HUM ELIMINATOR AND HEART RATE DETECTION USING ELECTROCARDIOGRAPHY

Hum noise created by poor power suppliers, transformers, or electromagnetic interference sourced by a main power supply is characterized by a frequency of 60 Hz and its harmonics. If this noise interferes with a desired audio or biomedical signal (e.g., in electrocardiography [ECG]), the desired signal could be corrupted. The corrupted signal is useless without signal processing. It is sufficient to eliminate the 60-Hz hum frequency with its second and third harmonics in most practical applications. We can complete this by cascading with notch filters having notch frequencies of 60 Hz, 120 Hz, and 180 Hz, respectively. Figure 8.41 depicts the functional block diagram.

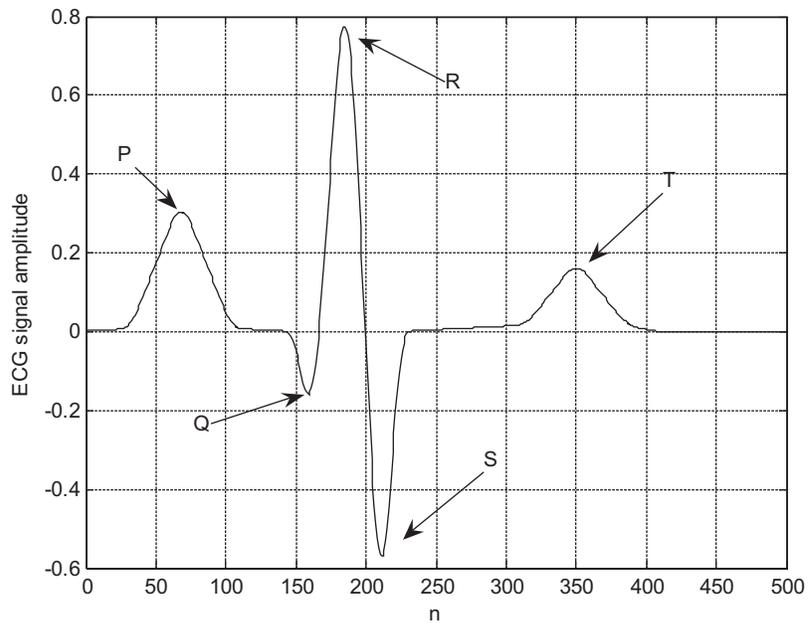
Now let us apply the 60-Hz hum eliminator to an ECG recording system. ECG is a small electrical signal captured from an ECG sensor. The ECG signal is produced by the activity of the human heart, thus it can be used for heart rate detection, fetal monitoring, and diagnostic purposes. The single pulse of the ECG is depicted in Figure 8.42, which shows that the ECG signal is characterized by five peaks and valleys, labeled P, Q, R, S, and T. The highest positive wave is the R wave. Shortly before and after the R wave are negative waves called the Q wave and S wave. The P wave comes before the Q wave, while the T wave comes after the S wave. The Q, R, and S waves together are called the QRS complex.

The properties of the QRS complex, with its rate of occurrence and times, heights, and widths, provide information to cardiologists concerning various pathological conditions of the heart. The



**FIGURE 8.41**

(Top) 60-Hz hum eliminator; (middle) the filter frequency response of the eliminator; (bottom) the input signal spectrum corrupted by the 60-Hz hum and its second and third harmonics.



**FIGURE 8.42**

The characteristics of the ECG pulse.

reciprocal of the time period between R wave peaks (in milliseconds) multiplied by 60,000 gives instantaneous heart rate in beats per minute. On a modern ECG monitor, the acquired ECG signal is displayed for diagnostic purposes.

However, a major source of frequent interference is the electric-power system. Such interference appears on the recorded ECG data due to electrical-field coupling between the power lines and the electrocardiograph or patient, which is the cause of the electrical field surrounding power lines (mains). Another cause is magnetic induction in the power line, whereby current in the power line generates a magnetic field around the line. Sometimes, the harmonics of 60-Hz hum exist due to the nonlinear sensor and signal amplifier effects. If such interference is severe, the recorded ECG data becomes useless.

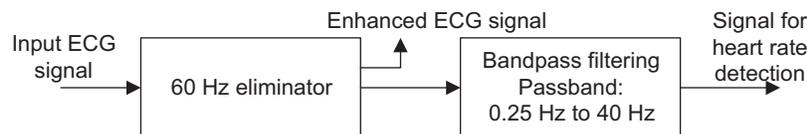
In this application, we focus on ECG enhancement for heart rate detection. To significantly reduce 60-Hz interference, we apply signal enhancement to the ECG recording system, as shown in Figure 8.43.

The 60-Hz eliminator removes the 60-Hz interference and has the capability to reduce its second harmonic of 120 Hz and third harmonic of 180 Hz.

The next objective is to detect the heart rate using the enhanced ECG signal. We need to remove DC drift and to filter muscle noise, which may occur at approximately 40 Hz or more. If we consider the lowest heart rate as 30 beats per minute, the corresponding frequency is  $30/60 = 0.5$  Hz. Choosing a lower cutoff frequency of 0.25 Hz should be reasonable.

Thus, a bandpass filter with a passband from 0.25 Hz to 40 Hz (range from 0.67 Hz to 40 Hz, discussed in Webster [1998]), either FIR or IIR type, can be designed to reduce such effects. The resultant ECG signal is valid only for the detection of heart rate. Notice that the ECG signal after bandpass filtering with a passband from 0.25 Hz to 40 Hz is no longer valid for general ECG applications, since the original ECG signal occupies the frequency range from 0.01 Hz to 250 Hz (diagnostic-quality ECG), as discussed in Carr and Brown (2001) and Webster (1998). The enhanced ECG signal from the 60-Hz hum eliminator can serve for general ECG signal analysis (which is beyond the scope of this book). We summarize the design specifications for the heart rate detection application as follows:

System outputs:	Enhanced ECG signal with 60-Hz elimination Processed ECG signal for heart rate detection
60-Hz eliminator specifications:	
Harmonics to be removed:	60 Hz (fundamental) 120 Hz (second harmonic) 180 Hz (third harmonic)
3-dB bandwidth for each filter:	4 Hz
Sampling rate:	600 Hz
Notch filter type:	Second-order IIR



**FIGURE 8.43**

ECG signal enhancement system.

Design method:	Pole-zero placement
Bandpass filter specifications:	
Passband frequency range:	0.25–40 Hz
Passband ripple:	0.5 dB
Filter type:	Chebyshev fourth order
Design method:	Bilinear transformation method
DSP sampling rate:	600 Hz

Let us carry out the 60-Hz eliminator design and determine the transfer function and difference equation for each notch filter and bandpass filter. For the first section with the notch frequency of 60 Hz, applying Equations (8.45) to (8.48) leads to

$$r = 1 - (4/600) \times \pi = 0.9791$$

$$\theta = \left(\frac{60}{600}\right) \times 360^\circ = 36^\circ$$

We calculate  $2 \cos(36^\circ) = 1.6180$ ,  $2r \cos(36^\circ) = 1.5842$ , and

$$K = \frac{(1 - 2r \cos \theta + r^2)}{(2 - 2 \cos \theta)} = 0.9803$$

Hence it follows that

$$H_1(z) = \frac{0.9803 - 1.5862z^{-1} + 0.9803z^{-2}}{1 - 1.5842z^{-1} + 0.9586z^{-2}}$$

$$y_1(n) = 0.9803x(n) - 1.5862x(n-1) + 0.9802x(n-2) + 1.5842y_1(n-1) - 0.9586y_1(n-2)$$

Similarly, we obtain the transfer functions and difference equations for the second section and third section as follows:

Second section:

$$H_2(z) = \frac{0.9794 - 0.6053z^{-1} + 0.9794z^{-2}}{1 - 0.6051z^{-1} + 0.9586z^{-2}}$$

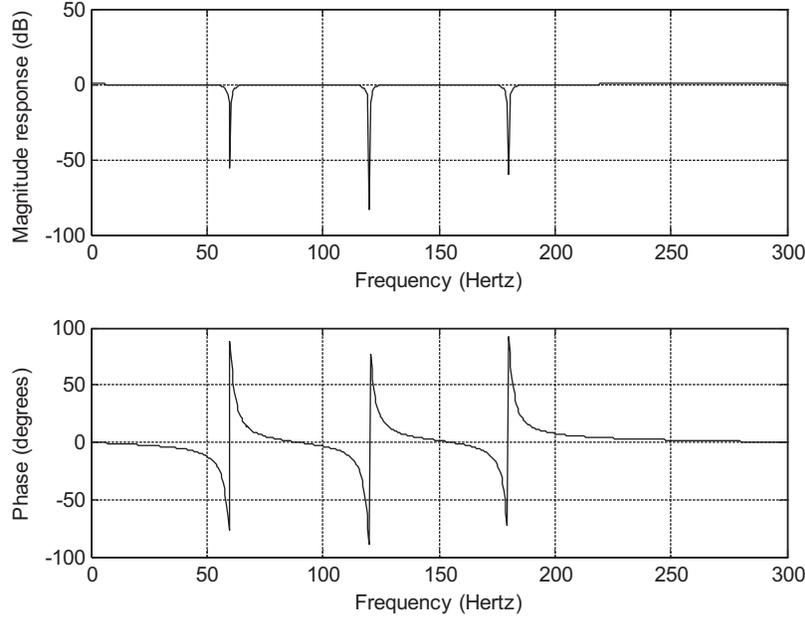
$$y_2(n) = 0.9794y_1(n) - 0.6053y_1(n-1) + 0.9794y_1(n-2) + 0.6051y_2(n-1) - 0.9586y_2(n-2)$$

Third section:

$$H_3(z) = \frac{0.9793 + 0.6052z^{-1} + 0.9793z^{-2}}{1 + 0.6051z^{-1} + 0.9586z^{-2}}$$

$$y_3(n) = 0.9793y_2(n) + 0.6052y_2(n-1) + 0.9793y_2(n-2) - 0.6051y_3(n-1) - 0.9586y_3(n-2)$$

The cascaded frequency responses are plotted in Figure 8.44. As we can see, the rejection for each notch frequency is below 50 dB.



**FIGURE 8.44**

Frequency responses of three cascaded notch filters.

The second-stage design using the BLT gives the bandpass filter transfer function and difference equation:

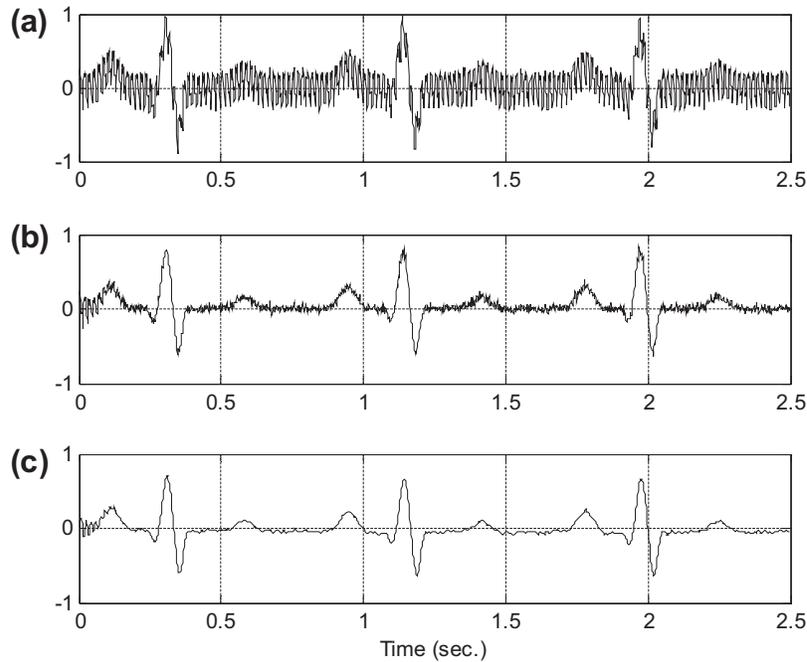
$$H_4(z) = \frac{0.0464 - 0.0927z^{-2} + 0.0464z^{-4}}{1 - 3.3523z^{-1} + 4.2557z^{-2} - 2.4540z^{-3} + 0.5506z^{-4}}$$

$$y_4(n) = 0.0464y_3(n) - 0.0927y_3(n-2) + 0.0464y_3(n-4) \\ + 3.3523y_4(n-1) - 4.2557y_4(n-2) + 2.4540y_4(n-3) - 0.5506y_4(n-4)$$

Figure 8.45 depicts the processed results at each stage. In Figure 8.45, plot (a) shows the initial corrupted ECG data, which includes 60-Hz interference and its 120 and 180 Hz harmonics, along with muscle noise. Plot (b) shows that the 60-Hz interference and its harmonics of 120 and 180 Hz have been removed. Finally, plot (c) displays the result after the bandpass filter. As we expected, the muscle noise has been removed; and the enhanced ECG signal is observed. A MATLAB simulation is provided in Program 8.16.

With the processed ECG signal, a simple zero-cross algorithm can be designed to detect the heart rate. Based on plot (c) in Figure 8.45, we use a threshold value of 0.5 and continuously compare each of two consecutive samples with the threshold. If both results are opposite, then a zero crossing is detected. Each zero-crossing measure is given by

$$\text{zero crossing} = \frac{|cur\_sign - pre\_sign|}{2}$$



**FIGURE 8.45**

Results of ECG signal processing. (a) Initial corrupted ECG data; (b) ECG data enhanced by removing 60-Hz interference; (c) ECG data with DC blocking and noise removal for heart rate detection.

where *cur\_sign* and *pre\_sign* are determined based on the current input  $x(n)$ , the past input  $x(n - 1)$ , and the threshold value, given as

$$\text{if } x(n) \geq \text{threshold } \text{cur\_sign} = 1 \text{ else } \text{cur\_sign} = -1$$

$$\text{if } x(n - 1) \geq \text{threshold } \text{pre\_sign} = 1 \text{ else } \text{pre\_sign} = -1$$

Figure 8.46 summarizes the algorithm.

After detecting the total number of zero crossings, the number of peaks will be half the number of zero crossings. The heart rate in terms of pulses per minute can be determined by

$$\text{Heart rate} = \frac{60}{\left(\frac{\text{Number of enhanced ECG data}}{f_s}\right)} \times \left(\frac{\text{zero - crossing number}}{2}\right)$$

In our simulation, we have detected 6 zero-crossing points using 1,500 captured data points at a sampling rate of 600 samples per second. Hence,